Exercise solutions: belemnite stretch at Morcles Nappe

The problem is to determine the maximum, S_1, and minimum, S_2, stretches in the plane of the measurements, that is the plane of the slaty cleavage. The relationship between the principal stretches and the stretch of a particular material line, S_i, oriented at angle, α_i, is:

$$\left(\frac{\cos\alpha_i}{S_1}\right)^2+\left(\frac{\sin\alpha_i}{S_2}\right)^2=\left(\frac{1}{S_i}\right)^2, \quad i=1,n \tag{1}$$

In order to compute S_1 and S_2, measurements of S_i and α_i for stretched belemnites with two or more distinct orientations are used to solve n versions of (1) simultaneously. Hossain (1979) found a "best fit" to the large and scattered data set, assuming that all the data represent a single state of strain. In other words it was assumed the strain was homogeneous across the outcrops that yielded the measurements.

1) Rearrange (1) into the form of a straight line:

$$y_i = mx_i + b \tag{2}$$

Dividing through by cos a_i and rearranging, we have:

$$\left(\frac{1}{S_i\cos\alpha_i}\right)^2=\left(\frac{1}{S_1}\right)^2+\left(\frac{1}{S_2}\right)^2\tan^2\alpha_i, \quad i=1,n \tag{3}$$

Here the y-intercept is $\left(1/S_1\right)^2$ and the slope is $\left(1/S_2\right)^2$.

2) Read the data in belemnite.txt and recreate Hossain's Figure 5 using MATLAB.

The data are read and plotted using the following MATLAB m-script:

```
data = load('belemnite.txt');
```

3) Calculate the best fitting straight line to the belemnite data set. Write down the parameters of this line and compute the principal stretch magnitudes. Hossain estimated that $S_1 = 2.7$ and $S_2 = 1.35$. How do your results compare to his? Describe this deformation.

```
A = [ones(size(data(:,1))) data(:,1)];
y = data(:,2);
x = (A'*A)\A'*y;
s1 = sqrt(1/x(1))
s2 = sqrt(1/x(2))
```

The computed values of stretch are $S_1 = 2.36$ and $S_2 = 1.44$. This deformation corresponds to extension in both directions within the plane that contains the belemnite with the S_1 component being approximately 1.6 times greater (compared to 2 times greater for Hossain's values).

 © David D. Pollard and Raymond C. Fletcher 2005

4) Use your estimates of the principal stretches to compute the ratio of the final area to initial area in the plane of the slaty cleavage. What can you say about the third principal stretch in terms of its magnitude and direction?

$$\frac{A_f}{A_i} = \frac{L_f(1)}{L_i(1)} \frac{L_f(2)}{L_i(2)} = S_1 S_2 = 3.4 \tag{4}$$

The orientations of S_1 and S_2 do not give any insight into the magnitude of S_3 without making any further assumptions. However, the third principal stretch is commonly oriented normal to the plane of slaty cleavage. Greatest shortening is expected to be normal to the plane of slaty cleavage, thus the stretch in the third principal stretch direction is expected to be smaller than 1.

5) Assume that volume was approximately unaltered by the deformation and that two of the principal stretches were in the plane of the measurements. Calculate the stretch S_3 normal to the plane of measurements and describe this component of the deformation. How might this relate to the slaty cleavage that defines the plane of the measurements?

$$V_i = L_i(1) L_i(2) L_i(3), \text{ and } V_f = L_f(1) L_f(2) L_f(3) \tag{5}$$

So, for $V_i = V_f$ we have:

$$\frac{V_f}{V_i} = \frac{L_f(1) L_f(2) L_f(3)}{L_i(1) L_i(2) L_i(3)} = S_1 S_2 S_3 = 1 \tag{6}$$

Rearranging (6) and substituting from (4):

$$S_3 = \frac{1}{S_1 S_2} \approx 0.3 \tag{7}$$

The stretch S_3 is a shortening to about 30% of the initial value. Cleavage planes are likely formed in a plane directed normal to the direction of greatest shortening and contains the direction in which the rock has undergone the greatest extension. Presumably this shortening caused a rotation of the clay minerals that now defines the slaty cleavage.

6) Evaluate the assumption that the deformation in the vicinity of a deformed belemnite is homogeneous. Use the outcrop photograph from Hossain (1992) and the sketch (Figure 1) to support your evaluation.

Both the natural example (Hossain, Figure 1) and the sketch given here (Figure 1) demonstrate that the belemnite broke apart during the stretching. It appears that two stages of stretching occurred, first with calcite deposition and later with quartz deposition in the opening cavity. The cavities do not extend into the surrounding slate so this deformation is not, strictly speaking homogeneous. The assumption used here is that the stretch of the belemnite approximates the stretch in the surround slate even though it did not fracture.

 © David D. Pollard and Raymond C. Fletcher 2005

7) Briefly describe the design of either a laboratory or a numerical experiment to determine the principal stretches from "model" belemnites under conditions of heterogeneous deformation.

Lab experiments designed to test the use of nearly rigid broken particles as a proxy for strain markers are shown in the paper by Hussain (1979, Figure 2a,b). These experiments demonstrate that the rigid blocks rotate when the principal stretch axes are oblique to the long axis of the model belemnite. This type of rotation was observed in the field examples. These experiments do not quantify the relationship between the apparent stretch of the model belemnite and the average stretch of the surrounding putty.

 © David D. Pollard and Raymond C. Fletcher 2005