

## Exercise: concepts from chapter 6

Reading: Fundamentals of Structural Geology, Chapter 6

- 1) The definition of the traction vector (6.7) relies upon the approximation of rock as a continuum, so the ratio of resultant force to surface area approaches a definite value at every point. Use the model of rock as a granular material suggested by Amadei and Stephansson (1997) and (6.15) to compute multiple realizations of the ratio  $t_p/t$  and recreate a figure similar to Figure 6.7b. Choose a mean and standard deviation for grain/pore widths based on the geometry of the sandstone in Figure 1 and use these to define a normal distribution of widths. A representative spring constant for quartz grains is  $6 \times 10^7$  N/m, and for the pores the spring constant is zero.

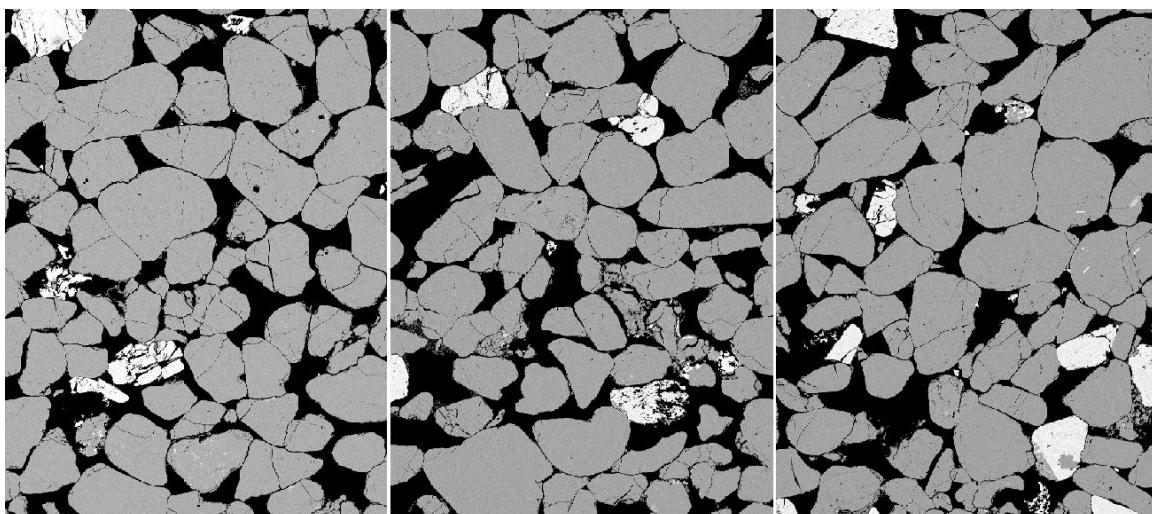


Figure 1. SEM images of Aztec sandstone, Valley of Fire, NV. (Sternlof, Rudnicki and Pollard, 2005). The base of each image is approximately 1.5mm wide; the gray shapes are grains of quartz; the black shapes are pores.

- 2) Slip on faults may be thought of as driven by the shear component,  $t_s(\mathbf{n})$ , and resisted by the (negative) normal component,  $t_n(\mathbf{n})$  of the traction vector  $\mathbf{t}(\mathbf{n})$  acting on the fault surface. Consider the surface of a vertical strike-slip fault oriented with outward unit normal,  $\mathbf{n}$ , in the direction  $350^\circ$ . Choose Cartesian coordinates,  $x$  and  $y$ , in the horizontal plane in the directions  $100^\circ$  and  $010^\circ$  respectively. Consider a right triangular free body (a 2D analogue to the Cauchy tetrahedron as shown in Fig. 6.8) with hypotenuse parallel to the fault, side a with outward normal in the positive  $x$ -direction, and side b with outward normal in the negative  $y$ -direction. We postulate that no shear tractions act on sides a and b, and that the normal components of the tractions on them are  $t_x(a) = -425$  MPa and  $t_y(b) = +135$  MPa respectively.

- a) Draw a carefully labeled sketch map of the fault surface with normal vector  $\mathbf{n}$ , the triangular free body, the traction vector components  $t_x(a)$  and  $t_y(b)$  acting on this free body, a north arrow, and the Cartesian coordinate system.
  - b) Calculate the Cartesian components of the traction vector  $\mathbf{t}(\mathbf{n})$  starting with the fundamental equations for balancing forces on the triangular free body in the two coordinate directions. Include these components on your sketch map.
  - c) Calculate the magnitude and direction of the traction vector  $\mathbf{t}(\mathbf{n})$ . Determine the magnitude and direction of the traction vector acting on the adjacent fault surface with outward unit normal  $-\mathbf{n}$ .
  - d) Calculate the normal and shear traction components acting on the fault surface with outward normal  $\mathbf{n}$ . Draw these traction components on another sketch map of the fault surface. Indicate if you would expect slip resulting from the traction to be left- or right-lateral and show this on your sketch map with appropriate arrows.
- 3) We postulate the following stress state in the horizontal plane of the outcrop pictured in Figure 2 at the time of motion on the fault:

$$\sigma_{xx} = -40 \text{ MPa}, \sigma_{xy} = -60 \text{ MPa} = \sigma_{yx}, \sigma_{yy} = -100 \text{ MPa} \quad (1)$$

Note that a Cartesian coordinate system is prescribed along with the outward unit normal vector,  $\mathbf{n}$ , for one surface of the fault. Also note that both normal stress components are negative (compressive) and the shear stress is negative.



Figure 2. Outcrop from the Lake Edison granodiorite of the Sierra Nevada, CA with a small left-lateral fault (Segall & Pollard, 1983). The unit normal vector  $\mathbf{n}$  is positioned on the fault surface that bounds the block of rock in the lower right hand part of the photograph and points outward from this block. In what follows we will be seeking the

traction vector that represents the mechanical action of the block in the upper left hand part of the photograph on the block in the lower right hand part.

- a) Find the components of the outward unit normal vector,  $n_x$  and  $n_y$ , in the prescribed coordinate system and use them to compute  $t_x(\mathbf{n})$  and  $t_y(\mathbf{n})$ , the  $x$ - and  $y$ -components of the traction vector acting on the fault surface. The direction angles can be measured on Figure 2 with a protractor.
  - b) Calculate  $t(\mathbf{n})$  and  $\alpha(\mathbf{n})$ , the magnitude and direction of the traction,  $\mathbf{t}(\mathbf{n})$ , acting on the fault surface with outward normal  $\mathbf{n}$ . Remember to use the proper inverse tangent function in Matlab for the direction.
  - c) Draw a carefully labeled sketch map of the fault surface with normal vector  $\mathbf{n}$ , the traction vector  $\mathbf{t}(\mathbf{n})$ , a north arrow, and the Cartesian coordinate system. Use your sketch to indicate if this traction pushes or pulls against that surface. Remember it represents the mechanical action of the rock in the upper left part of the photo.
  - d) Calculate the normal and shear traction components on the fault surface. Recall the right-hand rule for the normal and tangential axes. Is the sign of the shear traction component consistent with the sense of shearing indicated by the offset dike in Figure 2? Explain your reasoning.
- 4) Any vector,  $\mathbf{v}$ , may be resolved into two vector components that are, respectively, parallel and perpendicular to an arbitrary unit normal vector,  $\mathbf{n}$  (Malvern, 1969):
- $$\mathbf{v} = (\mathbf{v} \cdot \mathbf{n})\mathbf{n} + \mathbf{n} \times (\mathbf{v} \times \mathbf{n}) \quad (2)$$
- This vector equation has important applications in structural geology where the vector in question is the traction,  $\mathbf{t}$ , acting on a geological surface with outward normal  $\mathbf{n}$ .
- a) Use a carefully labeled sketch of the Cauchy tetrahedron to illustrate the application of (2) to the traction vector acting on a planar surface of arbitrary orientation. Show the Cartesian coordinate system, the normal vector, the traction vector, and the normal and shear vector components of the traction with their magnitudes.
  - b) Use the definition of the scalar product to write an equation for the scalar component,  $t_n$ , of  $\mathbf{t}$  acting parallel to  $\mathbf{n}$  in terms of the angle  $\theta$  between these two vectors. Illustrate this relationship on your sketch of the Cauchy tetrahedron. Describe how this scalar component informs one whether the traction pulls or pushes on the prescribed surface. Write this scalar component in terms of the Cartesian components of  $\mathbf{t}$  and  $\mathbf{n}$ , and use this result to write an equation for  $\mathbf{t}_n$ , the vector component of  $\mathbf{t}$  acting parallel to  $\mathbf{n}$ .
  - c) Use the definition of the magnitude of a vector product to derive an equation for the magnitude,  $|t_s|$ , of the scalar component of  $\mathbf{t}$  acting perpendicular to  $\mathbf{n}$  in terms of the angle  $\theta$  between these two vectors. Illustrate this relationship on your sketch of the Cauchy tetrahedron. This is the component of the traction taken

tangential to the plane of the surface, so it is referred to as the shear component. Explain why the sign of the shear component is ambiguous. Use the second term on the right side of (2) to write an equation for  $\mathbf{t}_s$ , the vector component of  $\mathbf{t}$  acting perpendicular to  $\mathbf{n}$  in terms of the Cartesian components of  $\mathbf{t}$  and  $\mathbf{n}$ . In doing so explain geometrically what is meant by the two cross products. Use your sketch to derive a simple equation for the magnitude of the shear component in terms of  $\mathbf{t}$  and  $\mathbf{t}_n$ .

5) Write a MATLAB script that takes as input the six independent components of the stress tensor associated with a given Cartesian coordinate system and a homogeneous stress field. Provide as output the following quantities:

- a) the components ( $t_x, t_y, t_z$ ) of the traction vector,  $\mathbf{t}$ , acting on surfaces with outward unit normal vector  $\mathbf{n}$  defined by its components ( $n_x, n_y, n_z$ );
- b) the magnitude of  $\mathbf{t}$  and the direction of  $\mathbf{t}$  in terms of direction cosines;
- c) the magnitude of the shear component,  $|t_s|$ , and the normal component,  $t_n$ , of the traction vector;
- d) the direction of the shear component in terms of direction cosines;
- e) the principal values of the stress tensor and the direction cosines for the principal directions; and
- f) the traction ellipsoid corresponding to this state of stress.

6) Consider the elastic stress field within a circular disk loaded by opposed point forces (Figure 3). The two-dimensional solution is given in equations (6.97 – 6.99).

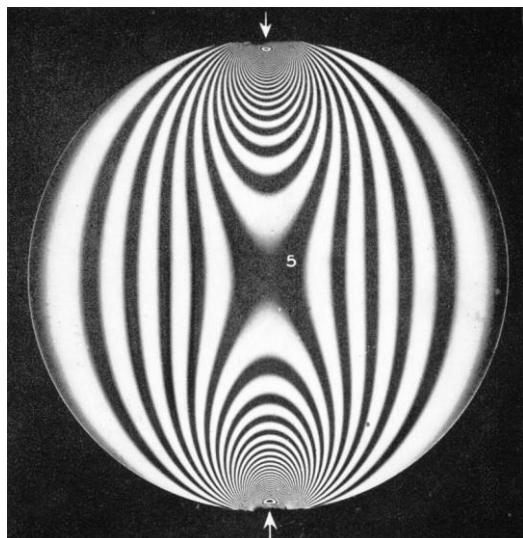


Figure 3. Photoelastic image of the maximum shear stress field in a circular disc loaded by diametrically opposed forces (Frocht, 1948).

- a) Write a MATLAB script to generate contour plots of the principal stress magnitudes and the maximum shear stress magnitude in the circular disk. Compare your plot of the maximum shear stress distribution to the photoelastic image in Figure 3. Frocht concluded from his experimental and theoretical results that, “Inspection of these curves shows the corroboration to be well-nigh perfect.”
- b) Using equations (6.97 – 6.99) describe what happens to the stress components near the point of application of the point forces according to the elastic theory. Indicate whether you think this state of stress can be realized in the photoelastic experiment. Suggest what happens near these points in the experiment considering both the nature of the implement used to apply the loads and the nature of the material making up the disc.
- c) Compare your plot of the maximum shear stress to the photoelastic images of model sand grains in Figure 6\_frontispiece. Pick a couple of examples that are similar and a couple that are different and explain why.
- d) Use the distribution of principal stresses to deduce where tensile cracks might initiate in the disk and in what direction those cracks might propagate. Explain your reasoning. Point out a couple of examples of cracks within the sand grains of Figure 1 that are consistent with your deductions, and examples that are not. Suggest some reasons why cracks in the sand grains might not be consistent with your deduction from the elastic theory.
- 7) The polar stress components given in equations (6.108 – 6.110) solve the two-dimensional elastic boundary value problem of a pressurized cylindrical hole in a biaxial remote stress state. This solution was published by G. Kirch in 1898 and has found many applications in structural geology and rock mechanics. Figure 4 shows the distribution of maximum shear stress induced by a remote uniaxial tension of unit magnitude.

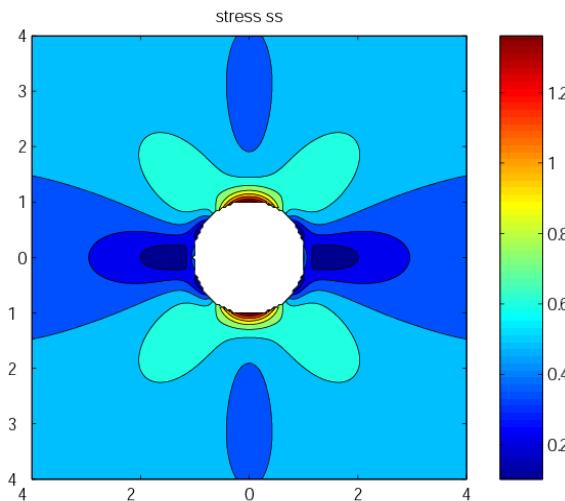


Figure 3. Contour map of maximum shear stress around a cylindrical hole with uniaxial remote tension. Note the stress concentrations and diminutions on the edge of the hole.

Here we use the Kirsh solution to address some practical problems related to wellbore stability and hydraulic fracturing. Consider a vertical cylindrical hole (the model wellbore) loaded by an internal pressure of magnitude  $P$ , and remote horizontal principal stresses of magnitude  $S_H$  and  $S_h$  (Figure 6.33).

- a) Write a MATLAB script to calculate the polar stress components. Verify your script (at least in part...) by determining that the following boundary conditions are satisfied. A good way to accomplish this is to plot graphs similar to those in Figure 6.34a. Also, write down the remote boundary conditions for  $\theta = \pi/2$ .

$$\text{BC: for } 0 \leq \theta \leq 2\pi, \text{ as } r/R \rightarrow 1, \begin{cases} \sigma_{rr} \rightarrow -P \\ \sigma_{r\theta} \rightarrow 0 \end{cases} \quad (3)$$

$$\text{BC: for } \theta = 0, \text{ as } r/R \rightarrow \infty, \begin{cases} \sigma_{rr} \rightarrow -S_H \\ \sigma_{r\theta} \rightarrow 0 \\ \sigma_{\theta\theta} \rightarrow -S_h \end{cases} \quad (4)$$

- b) Write a MATLAB script to plot a graph, similar to Figure 6.34b. Begin by plotting the polar stress components for the same boundary conditions used in part a) and describe their variation. Then use this script to determine what internal fluid pressure,  $P$ , is required to exactly nullify (reduce to zero) the compressive stress concentration in the circumferential stress component,  $\sigma_{\theta\theta}$ , on the edge of the hole imposed by a remote lithostatic pressure  $P_L$ , where  $S_H = S_h = P_L$ . Plot the polar stress components for  $P_L = 2 \text{ MPa}$  and  $P = 1 \text{ MPa}$ . Define the relationship between the fluid pressure and the lithostatic pressure that marks the transition from a compressive to a tensile stress,  $\sigma_{\theta\theta}$ , at the wellbore.
- c) Predict where hydraulic fractures would initiate and where those fractures would propagate given the ratio of horizontal principal stresses,  $S_H / S_h = 3 / 2$ . Use the same remote stress conditions to predict where borehole breakouts would occur. Draw a sketch of each prediction including the wellbore and the horizontal principal stresses.
- d) The average unit weight of rock at a particular drilling site is  $2.5 \times 10^4 \text{ N/m}^3$  and the unit weight of the fluid in the wellbore is  $1.0 \times 10^4 \text{ N/m}^3$ . Calculate the circumferential stress at the wellbore at a depth of 1 km. Calculate the pressure that must be imposed at the wellhead to reach the transition state where the circumferential stress goes from compressive to tensile. Also calculate the wellhead pressure necessary to induce the hydraulic fracture at this depth.