Exercise: mechanics of dike formation at Ship Rock

Reading: Fundamentals of Structural Geology, Ch. 8, p. 287 – 295, 319 - 323
Delaney & Pollard, 1981, Deformation of host rocks and flow of magma during growth of minette dikes and breccia-bearing intrusions near Ship Rock, NM. USGS Professional Paper 1202, p. 13 – 22, 29 – 36

In this exercise you will explore models of rock deformation during the emplacement of dikes in the magma plumbing systems of volcanoes. Understanding the mechanics of dike formation is important because dikes provide the conduits for flow of magma through the brittle portion of Earth's crust and because dikes feed fissure eruptions on many volcanoes (Figure 1). Thus, knowledge of the formation of dikes contributes to geological investigations of the evolution of Earth's crust and to the mitigation of volcanic hazards.

Figure 1. Fissure eruption in the southwest rift zone of Kilauea Volcano, Hawaii, on December 31, 1974 (Pollard, Delaney, Duffield, Endo, and Okamura, 1983).

The mechanical models used here are based on *elasticity theory* and describe the opening of a dike and the deformation of the surrounding rock. The dike opening is driven by the magma pressure, P, and resisted by the remote compressive stress, S_r, acting across the dike plane. This stress is largely caused by the weight of the overlying rock. The magma pressure must exceed the compressive stress for the dike to open and the amount of opening is proportional to the difference ($P - S_r$). Opening also is resisted by the elastic stiffness of the rock adjacent to the dike. Here we take as a measure of stiffness the quantity $E/(1 - \nu^2)$ where E is Young's modulus and ν is Poisson's ratio. These are material properties of isotropic elastic solids that can be measured in laboratory experiments. Poisson's ratio is limited to the range $0 \le \nu \le \frac{1}{2}$ and Young's modulus for sedimentary rock typically falls within the range $10^2\ \mathrm{MPa} \le \mathrm{E} \le 10^4\ \mathrm{MPa}$. We compare the opening of the model dike (Figure 2) to the distribution of thickness measured along the northeastern dike at Ship Rock, New Mexico and evaluate the roles of magma pressure, host rock stiffness, and compressive stress in the formation of this dike.

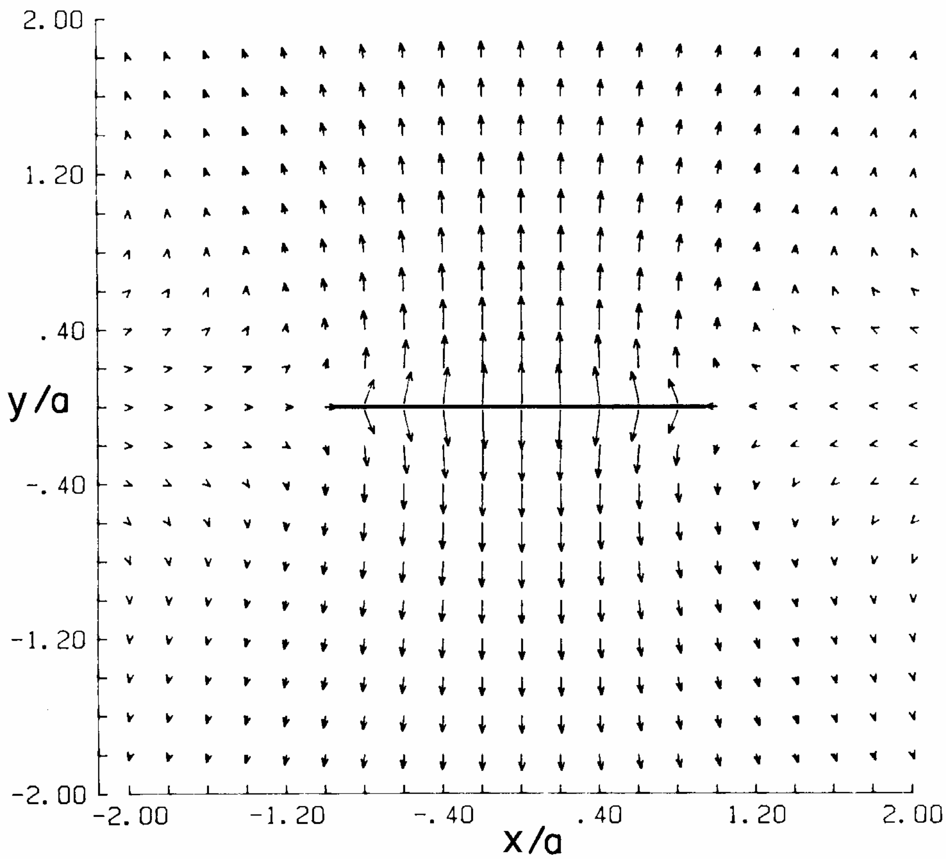

Figure 2. Displacement vector field for the opening of a model dike of length 2a with uniform magma pressure, *P*, in a rock mass with homogeneous elastic stiffness and a uniform remote compressive stress, S_r, acting across the dike plane.

The Cartesian coordinate system (Figure 2) is centered at the middle of the model dike and the two surfaces of the dike before the magma is injected are defined in terms of these coordinates as:

$$-a \le x \le +a,\ y = 0^+ \text{ and } -a \le x \le +a,\ y = 0^- \tag{1}$$

The shorthand equivalent of (1) is: $x \le |a|,\ y = 0^{\pm}$. Note that the two surfaces are (nearly) coincident with the *x*-axis and they are straight line segments of length, 2*a*. This is a two-dimensional model so all cross sections parallel to the plane $z = 0$ are identical. The *z*-component of displacement is zero, and the two in-plane components are functions of *x* and *y* only:

$$u_x = u_x(x, y),\ u_y = u_y(x, y),\ u_z = 0 \tag{2}$$

1) Describe the displacement field illustrated in Figure 2 indicating what is consistent with your intuition about the opening of a pressurized crack, and what is not. In particular comment on: a) the symmetry of the displacement field; b) the fact that displacement vectors to either side of the model dike have both *x*- and *y*-components; c) the fact that the dike shortens as it opens; and d) the spatial rate of decrease in magnitude of the displacement vectors along the coordinate axes.

2) Note that the displacement vectors associated with the two surfaces of the model dike in Figure 2 do not have the same directions. In fact there is a discontinuity in the displacement field such that the two surfaces move away from each other as the dike opens. Everywhere else the displacement is continuous and smoothly varying. The displacement discontinuity is defined for the two surfaces as a vector function of the *x*-coordinate over the length of the model dike:

$$\mathbf{D} = \mathbf{u}\left(x \le |a|, y = 0^+\right) - \mathbf{u}\left(x \le |a|, y = 0^-\right) \tag{3}$$

What is the *x*-component of **D** as shown in Figure 2, and how can you justify this in terms of the symmetry of loading of the dike? How is the *y*-component of **D** related to the dike thickness?

3) For the model dike of length 2*a* subject to uniform remote compressive stress, S_r, and uniform magma pressure, *P*, elasticity theory provides the following distributions for the components of the displacement discontinuity:

$$D_x = 0,\ D_y = 4\left(P - S_r\right)\left(\frac{1-\nu^2}{E}\right)\left(a^2 - x^2\right)^{1/2} \tag{4}$$

For soft sandstone and shale typical of the rocks in the Ship Rock region Poisson's ratio is small so we use the approximation $1-\nu^2 \approx 1$. A representative estimate for Young's modulus of these rocks is $\mathrm{E} \approx 4\times10^3$ MPa. The difference ($P - S_r$) is referred to as the driving stress for dike opening and an estimate for the northeastern dike at Ship Rock is $\left(P - S_r\right) \approx 2$ MPa. The length of the dike projected onto the local *x*-axis is 2*a* = 2901 m. Use these estimates of the physical quantities to plot a graph of D_y versus *x* using (4) and MATLAB. Describe the shape of the model dike under these conditions. If you change the elastic properties or the driving pressure does the shape change or just the thickness? Rearrange (4) to justify your answer.

4) Now compare the thickness of the northeastern dike at Ship Rock to the opening displacement discontinuity of the model dike. Continue your MATLAB m-file and read the text file 'contact.txt'. On a new figure plot the apparent thickness distribution for the northeastern dike using unconnected symbols. On the same figure plot the model dike opening displacement discontinuity as a continuous curve using (4) and adjust the elastic

stiffness to achieve a best fit to these data by eye. Compare your plot to that in Figure 3 from USGS Professional Paper 1202. What is your estimate of the elastic stiffness for the host rock at Ship Rock? The model significantly underestimates some data and significantly overestimates other data. Suggest reasons for both of these departures from the model dike.

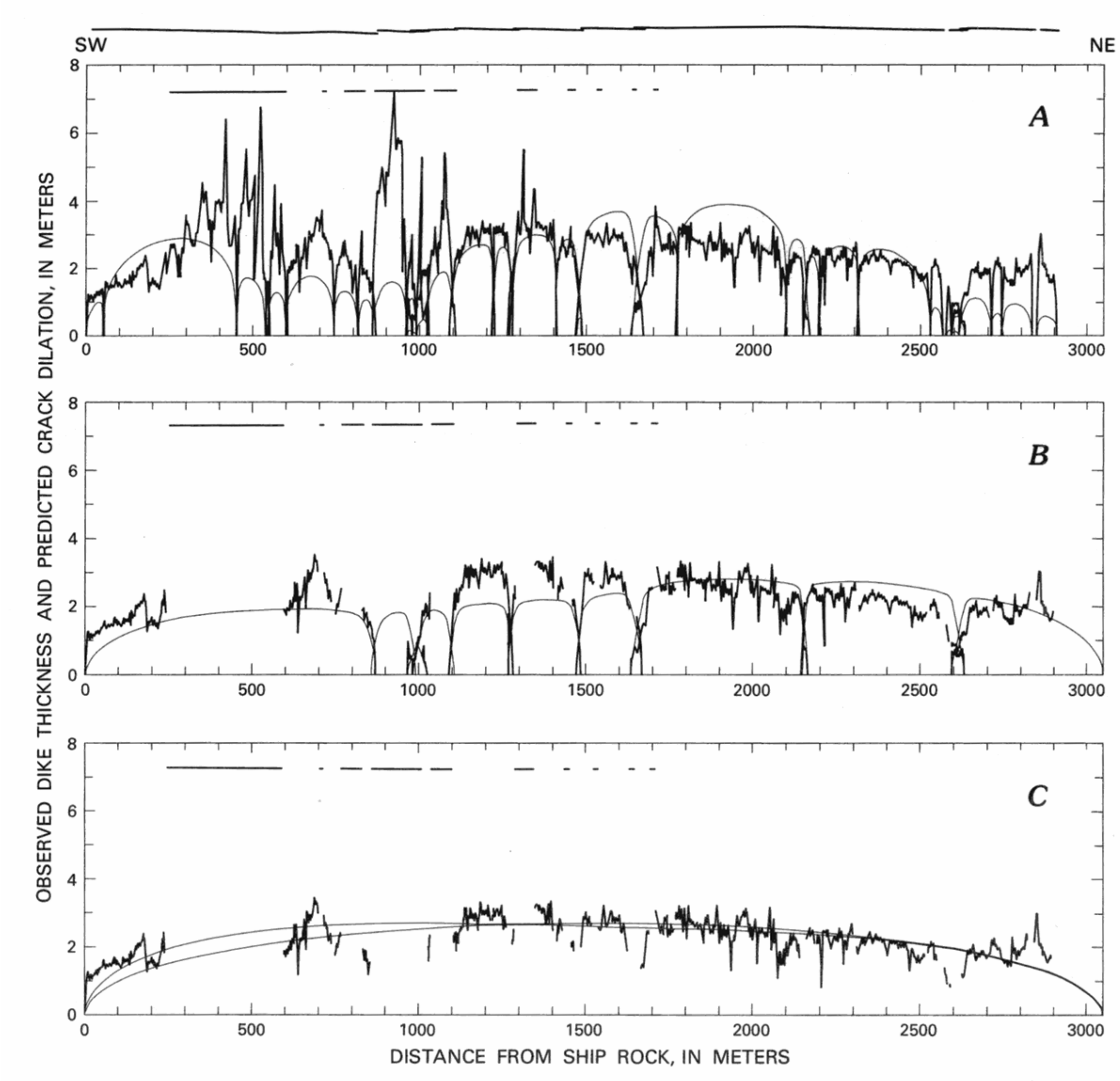

Figure 3. Observed dike thickness (jagged curve) and predicted crack dilation (smooth curve for three models using elasticity theory (Pollard and Delaney, 1981). A. all dike thickness data and model with 35 cracks. B. brecciated parts of dike removed and closely spaced segments joined, and model with 10 cracks. C. all segments joined and model with 1 crack.

5) Study the map of the northeastern dike and use your observations to decimate the data by removing stations that are not consistent with the opening of a single continuous fracture in an elastic material. You may erase these data from the text file or replace the values with NaN. Be sure to save the decimated data set as a tab delimited text file with a new file name. Continue your MATLAB m-file and read the new text file. On a new figure

plot for part 4) plot the apparent thickness distribution and the model dike opening displacement discontinuity, and adjust the elastic stiffness to achieve a best fit to these data by eye. What is your new estimate of the stiffness? List and briefly describe what you have ignored in making this estimate and point the way toward a better model.

6) Some of the northeastern dike segments have very little breccia and would appear to be good candidates for modeling as single opening fractures in an elastic material. For example consider the subset of data from 'contact.txt' for segment number 16 (Figure 4). Write a new MATLAB m-file and read the data for this segment. Translate the origin to the middle of this segment and plot the thickness distribution. How do the data near the two ends differ from the symmetric model? Discuss this asymmetry and suggest three mechanisms that could be responsible for it.

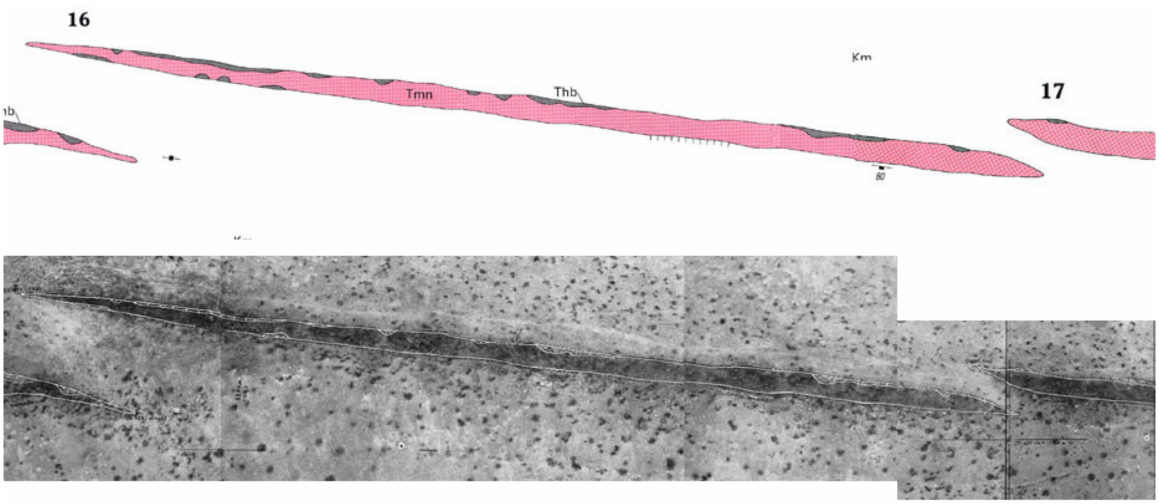

Figure 4. Northeastern Ship Rock dike segment 16.

7) To address asymmetric shape of dike segments we consider a linear variation in driving pressure, S_g, and elasticity theory provides the following components of the displacement discontinuity:

$$D_x = 0,\ \ D_y = 2\left(S_g\right)\left(\frac{1-\nu^2}{E}\right)x\left(a^2 - x^2\right)^{1/2} \qquad (5)$$

Use the approximation $1-\nu^2 \approx 1$; the estimate $E \approx 4\times10^3$ MPa ; the projected length 2a = 135m, and the driving pressure gradient $S_g = 1$ MPa/m to plot a graph of D_y versus x using (5). Describe the shape of the displacement discontinuity induced by the linear variation in driving pressure. What do negative values of D_y mean? Under what circumstances would negative values be physically realistic for a model dike?

8) Plot the data for dike segment 16 and a model curve for D_y that is the sum of (4) and (5). Adjust the driving pressure and the gradient to achieve the best fit by eye. Describe the misfit and suggest explanations based on the structural map of the northeastern dike.

9) To address the interaction of adjacent dike segments we consider a combination of a uniform driving stress, S_d, and a uniform compressive stress, S_c, acting across the ends of the model segment due to opening of the adjacent echelon segments. The size of the end zones are defined as $d \le |x| \le a$. The components of the displacement discontinuity are:

$$D_x = 0,\ D_y = \frac{4\left(1-\nu^2\right)}{\mathrm{E}} \left[\begin{array}{l} \left\{ \left(S_d - S_c\right) + S_c \dfrac{2}{\pi} \sin^{-1}\left(\dfrac{d}{a}\right) \right\} \left(a^2 - x^2\right)^{1/2} \\ + \dfrac{S_c}{\pi} \left\{ \begin{array}{l} \left(d + x\right) \cosh^{-1}\left(\dfrac{a^2 + dx}{a|x+d|}\right) + \\ \left(d - x\right) \cosh^{-1}\left(\dfrac{a^2 - dx}{a|x-d|}\right) \end{array} \right\} \end{array} \right] \qquad (6)$$

Use the approximation $1-\nu^2 \approx 1$; the estimate $\mathrm{E} \approx 4\times10^3$ MPa; the projected length $2a$ = 135m, and the driving and closing stress $S_d = 50$ MPa, $S_c = 200$ MPa/m to plot a graph of D_y versus x using (6). Describe the shape of the displacement discontinuity induced by this combination of stresses. Try other values of the stresses to eliminate negative values of D_y and gain some intuitive understanding for the shape of the model dike segment.

10) Plot the data for dike segment 16 and a model curve that is the sum of (5) and (6). Adjust the driving stress, stress gradient, and end-zone compressive stress to achieve the best fit by eye. Describe the misfit and suggest explanations based on the structural map.

