

Exercise: concepts from chapter 2

Reading: Fundamentals of Structural Geology, Ch 2

1) Develop a MATLAB script that plots the *spherical datum* (Fig. 2.1a) with unit radius as a wire-frame diagram using lines of constant latitude and longitude. An example is given here as Figure 1.

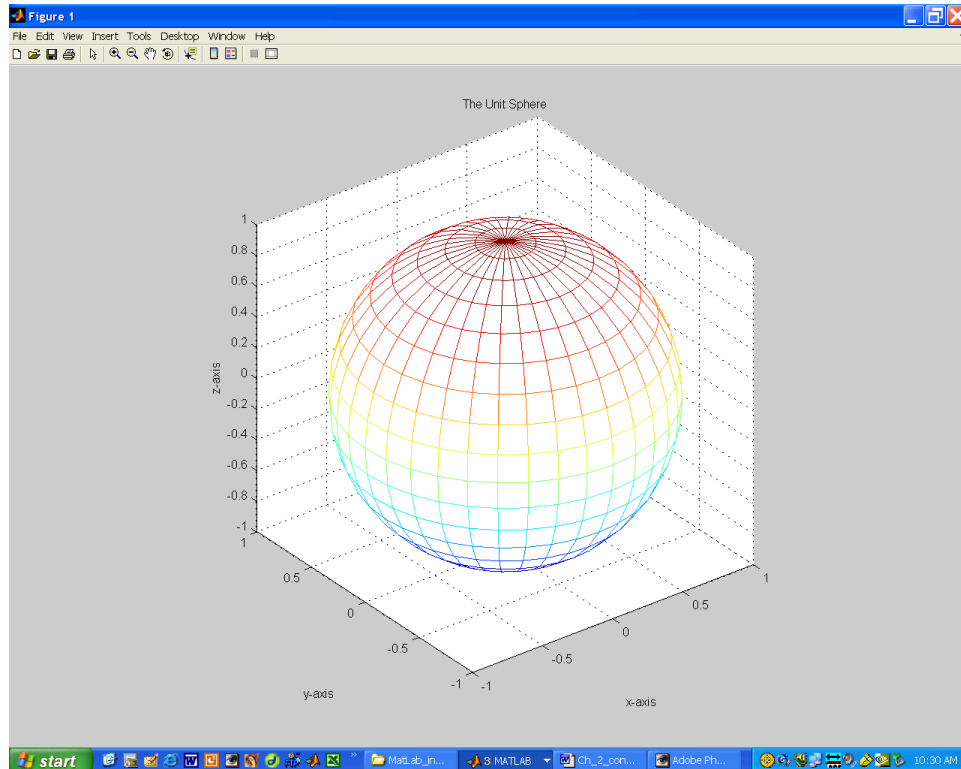


Figure 1. Matlab plot of the spherical datum.

2) The mathematical datum used for the *Global Positioning System* (GPS) is the *World Geodetic System* datum for 1984 (WGS-84). The standard physical datum, usually referenced to local mean sea level at the coastline, is the *geoid* (Fig. 2.2). Explain what is meant by a ‘datum’ and describe the general differences between WGS-84 and the geoid.

3) Investigate the *gnomonic projection* (Fig. 2.3) from a spherical datum with radius taken as the appropriate semi-axis of the GRS-80 ellipsoid. Consider two cases, $O(0, 0)$ and $O(\pi/2, 0)$, for the position of the origin $O(\phi_0, \lambda_0)$ where the projection plane is tangent to the sphere. Use MATLAB and equations (2.2) to plot lines of constant latitude and longitude in order to illustrate the distortion that is inherent to this projection. Describe these distortions.

4) Investigate the *transverse Mercator projection* (Fig. 2.4) from a spherical datum with radius taken as the appropriate semi-axis of the GRS-80 ellipsoid. Place the origin at the Greenwich meridian on the equator, $O(\varphi_0, \lambda_0) = O(0, 0)$, and let both the latitude and longitude extend $\pm\pi/4$ to either side of the origin. To accomplish this projection use the following equations (Bugayevskiy and Snyder, 1995, p. 158):

$$X = \frac{1}{2} R \ln \left(\frac{1 + \cos \varphi \sin \lambda}{1 - \cos \varphi \sin \lambda} \right) \quad (1)$$

$$Y = R \tan^{-1} (\tan \varphi \sec \lambda)$$

Note that equations (2.3) in the text plot the projection with origin at the pole, but X and Y are interchanged (see Errata). Use MATLAB and equations (1) to plot lines of constant latitude and longitude on the projection at the equator in order to illustrate the distortion that is inherent to it. Describe these distortions.

5) Most GPS receivers have onboard computers that are capable of reporting locations using the *Universal Transverse Mercator (UTM) projection* (Fig. 2.4). Explain, using a sketch, how the UTM projection is accomplished. Also, describe the UTM metric grid and how it is overlaid on this projection. In doing so explain what is meant by the following UTM coordinates of a location near Ship Rock, New Mexico, taken as the origin of a local coordinate system for mapping the northeastern dike (Fig. 2.5a):

Northern Hemisphere
Zone M12
Datum NAD-29
Easting: 694,000 m
Northing: 4,063,000 m
Elevation: 1,675 m

6) Rearrange the two-dimensional transformation equations for a rotation of axes about a common origin, as recorded in equations (2.13) to solve for the old coordinates, x and y , and derive equations (2.16). Draw a carefully labeled figure to identify the coordinates and angles, and show all the steps of the derivation.

7) Using the table of direction cosines (2.28) for the three dimensional transformation of coordinates, reduce the transformation equations to the two dimensional equivalent forms for a counterclockwise rotation through angle α about the z -axis in the (x, y) -plane (Fig. 2-8a). Some of the direction angles will have special values such as 0 or $\pi/2$: give the values for all direction angles and direction cosines. Draw a figure showing the relations among the direction angles and the angle α in the (x, y) -plane. Derive equations (2.13) from this more general three-dimensional form.

8) Consider the three dimensional rotational transformation of coordinates about the origin as described in the following table (2.27) for the position vector \mathbf{p} .

Fundamentals of Structural Geology
Exercise: concepts from chapter 2

$$\begin{array}{cccc}
 & p_{x'} & p_{y'} & p_{z'} \\
 p_x & m_{xx'} & m_{xy'} & m_{xz'} \\
 p_y & m_{yx'} & m_{yy'} & m_{yz'} \\
 p_z & m_{zx'} & m_{zy'} & m_{zz'}
 \end{array} \quad (2)$$

Here the vector components referred to the old (x, y, z) and the new (x', y', z') coordinate systems (Fig. 2.9a) are (p_x, p_y, p_z) and $(p_{x'}, p_{y'}, p_{z'})$ respectively, and the m_{ij} are the direction cosines of the direction angles between the i^{th} axis and the j^{th} axis.

- Write down the matrix of direction cosines corresponding to the interior of table (2) for a rotation of coordinates about the z -axis given by the angle ω_z . Confirm your result by comparison to the two-dimensional example (2.14).
- Compose a MATLAB script (m-file) that computes the vector components $(p_{x'}, p_{y'}, p_{z'})$ referred to the new coordinate system for a rotation of coordinates about the z -axis given by $\omega_z = \pi/6$. Define the position vector with components of unit value:

$$\mathbf{p} = p_x \mathbf{e}_x + p_y \mathbf{e}_y + p_z \mathbf{e}_z = \mathbf{1e}_x + \mathbf{1e}_y + \mathbf{1e}_z \quad (3)$$

- Continue developing your MATLAB script so that it transforms the components of \mathbf{p} found in part b) according to a rotation about the y' -axis by $\omega_{y'} = \pi/4$ to give new components $(p_{x''}, p_{y''}, p_{z''})$. Then transform these components according to a rotation about the x'' -axis by $\omega_{x''} = \pi/3$ to give components $(p_{x'''}, p_{y'''}, p_{z'''})$. For each transformation print the matrix of direction cosines corresponding to the interior of table (2) and print the components of the position vector after the final rotation.
- Calculate the matrix of direction cosines that accomplishes all three rotations of coordinates in one transformation. Write this matrix and the resulting vector components of \mathbf{p} and compare these to your result from part c). Now alter the order of rotation so you rotate first about the z -axis, then the x -axis and finally the y -axis. Compare the outcomes and comment upon the importance of order in these calculations.
- In general the rotation matrix is not symmetric: the transpose is not equal to the matrix itself. On the other hand the rotation is orthogonal: the transpose is equal to the inverse of the matrix. Show by example that the rotation matrix you have derived in part d) is not symmetric but is orthogonal. Recall that the product of a matrix and the inverse of that matrix yields the identity matrix.
- Demonstrate by calculation that the following equations (2.49) are true for the rotation matrix in part d).

$$m_{ki} m_{kj} = \delta_{ij}, \quad m_{ik} m_{jk} = \delta_{ij} \quad (4)$$

- Draw a sketch of the Cartesian coordinate system (x, y, z) and illustrate the positive sense of each rotation angle ($\omega_z = \pi/2$, $\omega_y = \pi/2$, $\omega_x = \pi/2$). Write down the general rule that describes this sense of rotation.

9) Figure 2.12 shows a dike segment from the northeast Ship Rock dike (Delaney and Pollard, 1981) and the elliptical coordinate system used in mechanical models of dikes.

- a) Use equations (2.32) to derive the standard equations for the set of ellipses and the set of orthogonal hyperbolae that define the elliptical coordinate system with common foci on the x -axis at $x = \pm f$ as pictured in Figure 2.12b.
- b) Derive the transformation equations for the elliptical coordinates of a point $P(\xi, \eta)$ given the Cartesian coordinates $P(x, y)$ of that point. Hint: consider the triangle formed by the line between the two foci and the line c drawing from any point $P(x, y)$ on the ellipse to the focus at $x = -f$ and the line d drawn to the focus at $x = +f$. Make two right triangles by dropping a perpendicular from $P(x, y)$ to the x -axis and solve for the lengths of the two sides, c and d , in terms of x and y . Then recall that for any ellipse the length of the semi-major axis is $a = (c+d)/2$. Use this and the standard equation for the ellipse from part a).
- c) Use MATLAB to plot a set of curves of constant ξ and a second set of curves of constant η to construct an illustration of the elliptical coordinate system (Fig. 2.12b).
- d) Describe the relationship between these curves and the lines of constant x and constant y for the Cartesian coordinate system in the limit as $\xi \rightarrow 0$, and in the limit as $\eta \rightarrow 0$.

10) On the equal-angle stereonet (Fig. 2.19) the orientation of planar elements may be defined by strike and dip (α_s, ϕ_d) and the orientation of linear elements lying in such a plane may be defined by the rake angle (θ_r). Derive the following equations for the azimuth of plunge and plunge angle (α_p, ϕ_p) of the linear element lying in the planar element:

$$\begin{aligned}\alpha_p &= \alpha_s + \tan^{-1} \left[(\sin \theta_r)(\cos \phi_d) / (\cos \theta_r) \right] \\ \phi_p &= \sin^{-1} \left[(\sin \theta_r)(\sin \phi_d) \right]\end{aligned}\tag{5}$$

Draw a carefully labeled diagram that shows all the angles and elements in relation to one another.

11) Use MATLAB to construct an equal-angle stereonet that includes the reference circle and the region inside the reference circle (e.g. Fig. 2.17b). An example stereonet is shown in Figure 2 below from the script `stereo_plot_ex11.m`. Download this script and study each line of code to understand how the stereonet is constructed from the equations presented in section 2.3.2 of the textbook. Use % at the beginning of lines to suppress computation and plotting of the sample point and great circle, and then run the modified code to construct and print the equal-angle stereonet for this exercise.

Plot the following planar and linear elements and determine the related angles using the stereonet. For the first pass through this exercise use a transparent overlay on your printed stereonet and construct the elements and angles by hand. For the second pass

through this exercise let MATLAB do the plotting of each linear and planar element. Label your plots carefully so each point and great circle are identified.

- a) Planar element with strike and dip (155, 72). What is the dip direction?
- b) Linear element with plunge direction and plunge (291, 24). What is the strike of a vertical planar element that contains this linear element? What is the rake of the linear element with reference to this strike direction? Plot the planar element.
- c) Normal to the planar element with strike and dip (155, 72). What are the plunge direction and plunge of the pole (normal) to the planar element?
- d) The rake is 142° for a linear element in the planar element with strike and dip (155, 72). What are the plunge direction and plunge of this linear element? Hint: equation (5) in question 10 above gives the azimuth of plunge direction and plunge for a linear element lying in a planar element.

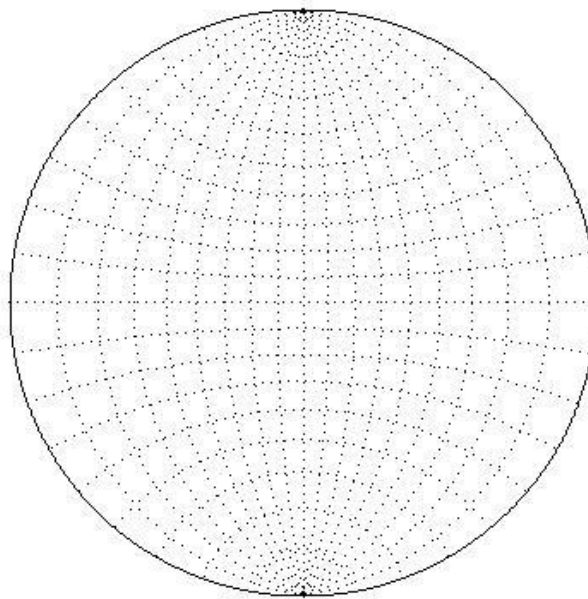


Figure 2. Equal-angle stereonet for region inside the reference circle.

12) You are given a Cartesian coordinate system with axes (x, y, z) that correspond to the Geographic coordinates (E, N, Up). Use MATLAB to calculate the direction cosines and direction angles for the normal to a planar element with strike and dip (026, 15)? Check your script by running the inverse transformation and recovering the strike and dip.

13) The GPS data for normal faults from the Chimney Rock area were gathered using a data dictionary that prompted the geologist to record the following information for each data station along the faults:

Easting Northing Elevation strike dip rake quality size formation (6)

Here strike and dip refer to the fault surface and rake refers to slickenlines on the fault surface. Two tab-delimited text files contain these data for the Blueberry fault and the Glass fault (numbers in parentheses are the number of data stations):

Blueberry_fault.txt	(92)
Glass_fault.txt	(60)

Write a MATLAB script that reads each text data file into a data array and parses the orientation data to extract local strike and dip (α_s , ϕ_d) of the faults. Plot the great circles representing these local orientations. Describe the orientations of the two faults, noting how consistent these are, and pointing out any outliers.

Convert the orientations into azimuth of plunge and plunge angle (α_p , ϕ_p) of the normals (poles) to the faults and plot these normals on a new stereonet.

Transform the azimuth of plunge and plunge angles (α_p , ϕ_p) to direction cosines and use these to compute the mean orientation for each fault. Write down the magnitude and components for the resultant vectors. Compare the vector magnitudes to the number of data stations for each fault. Plot the mean orientations on the stereographic projection along with all the normals.

Compute the spherical variance of the normals for the Blueberry fault and for the Glass fault. Evaluate your results indicating how tightly the data are clustered and how reliable the definition of each fault is, given the number of data stations, the spherical variance, and the number of outliers. Selectively remove outliers and recompute the spherical variances. Comment on your results and on the appropriateness of ignoring selected field data.